

How do wings work?

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Abstract

The popular explanation of lift is common, quick, sounds logical and gives the correct answer, yet also introduces misconceptions, uses a nonsensical physical argument and misleadingly invokes Bernoulli's equation. A simple analysis of pressure gradients and the curvature of streamlines is presented here to give a more correct explanation of lift.

M This article features online multimedia enhancements

The science behind aeronautics continues to fascinate and many students are attracted to engineering as a result of an early interest in aircraft. The most commonly asked question is how a wing can produce lift. Unfortunately the most widely used explanation of lift is wrong in a number of key points. Not only is this confusing for students, but in the worst case it can lead to a fundamental misunderstanding of some of the most important aerodynamic principles. In this article I will demonstrate why the popular explanation for lift is wrong and then propose an alternative explanation.

The popular explanation

Figure 1 shows a typical aerofoil—the cross-sectional shape of a wing—immersed in a flow where the streamlines have been visualized with smoke particles. At the front is the stagnation point (S), which is the location where the oncoming flow divides into that moving above and that moving below the wing. The argument revolves around the observation that the distance from this point S to the trailing edge (T) is greater along the upper surface than along the lower surface. If it is assumed that two neighbouring fluid particles which 'split' at S should meet again at T then this requires that the average velocity on the upper surface is greater than that on the lower surface.

Now Bernoulli's equation is quoted, which states that larger velocities imply lower pressures

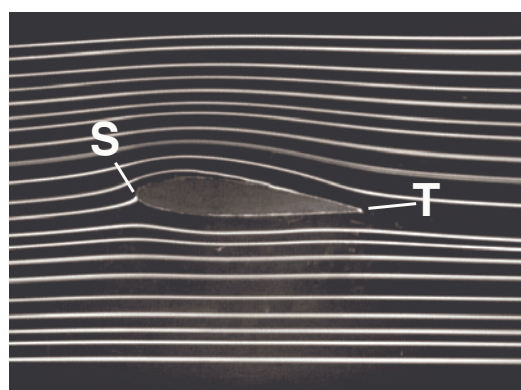


Figure 1. Streamlines around an aerofoil section visualized with smoke.

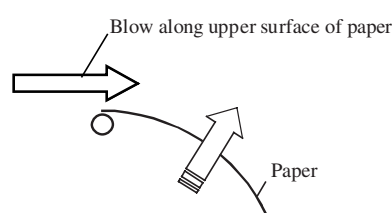


Figure 2. Paper lifts when air is blown along its upper surface.

and thus a net upwards pressure force is generated. Bernoulli's equation is often demonstrated by blowing over a piece of paper held between both hands as demonstrated in figure 2. As air is blown along the upper surface of the sheet of paper it rises

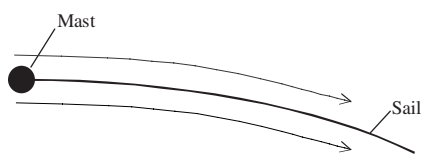


Figure 3. Flow along the cross section of a sail.

and, it is said, this is because the average velocity on the upper surface is greater (caused by blowing) than on the lower surface (where the air is more or less at rest). According to Bernoulli's equation this should mean that the pressure must be lower above the paper, causing lift.

The above explanation is extremely widespread. It can be found in many textbooks and, to my knowledge, it is also used in the RAF's instruction manuals. The problem is that, while it does contain a grain of truth, it is incorrect in a number of key places.

What's wrong with the 'popular' explanation?

The distance argument

While the aerofoil of figure 1 does indeed exhibit a greater distance between S and T along the upper surface, this is not a necessary condition for lift production. For example, consider a sail that is nothing but a vertical wing (generating side-force to propel a yacht). Figure 3 shows the cross section of a sail schematically and it is obvious that the distance between the stagnation point and the trailing edge is more or less the same on both sides. This becomes exactly true in the absence of a mast—and clearly the presence of the mast is of no consequence in the generation of lift. *Thus, the generation of lift does not require different distances around the upper and lower surfaces.*

The 'equal time' argument

It is often asked why fluid particles should meet up again at the trailing edge. Or, in other words, why should two particles on either side of the wing take the same time to travel from S to T? There is no obvious explanation and real-life observations prove that this is wrong. Figure 4 shows a selection of still frames from a video recording of a smoke-wind-tunnel experiment (the video clip is available in the online journal). Here, smoke particles are injected simultaneously upstream of a lifting aerofoil section, generating a line of smoke as seen in figure 4(a). This line of smoke moves with the flow, dividing into particles travelling above and below the aerofoil. By the time the smoke has passed the aerofoil (figure 4(c)) the particles moving along the upper surface are clearly ahead of those travelling along the lower surface. They do *not* meet up at the trailing edge. *When lift is generated, fluid particles travelling along the upper surface reach the trailing edge before those travelling along the lower surface*¹.

The Bernoulli demonstration

Blowing over a piece of paper does not demonstrate Bernoulli's equation. While it is true that a curved paper lifts when flow is applied on one side, this is not because air is moving at different speeds on the two sides. This can easily be demonstrated by blowing along one side of a *straight* piece of paper as sketched in figure 5. In this case the paper does not experience a force towards the side subjected to the faster moving air. The pressure on both sides of the paper is the

¹ In fact, it can be proved theoretically that if the particles on the upper surface reach the trailing edge at the same time as those travelling along the lower surface no lift is produced.

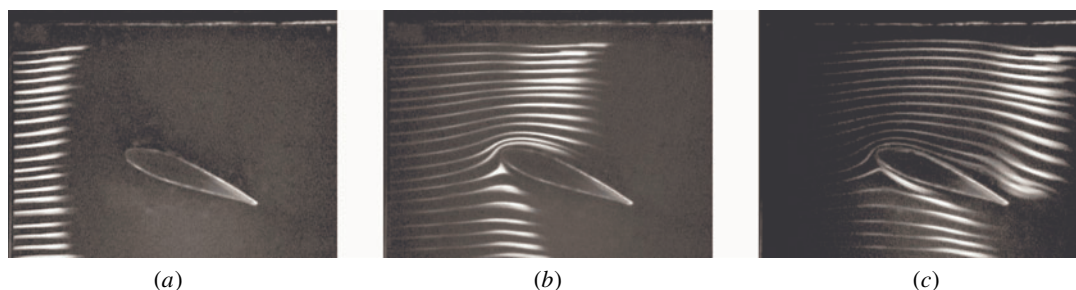


Figure 4. Smoke particles flowing along a lifting aerofoil section.

M An MPEG movie of this figure is available from stacks.iop.org/physed/38/497

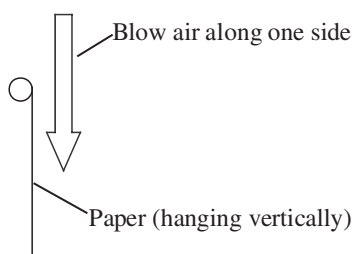


Figure 5. A straight piece of paper hanging vertically doesn't move when air is blown along one side.

same, despite the obvious difference in velocity. *It is false to make a connection between the flow on the two sides of the paper using Bernoulli's equation.*

An alternative explanation for lift

The above argument has remained popular because it is quick, sounds logical and gives the correct answer. However, my concern about using this explanation is that it introduces misconceptions about why aerofoil shapes generate lift, it uses a nonsensical physical argument and it often includes an erroneous 'demonstration' of Bernoulli's equation.

Before we begin—some basic assumptions

The key to understanding fluid flow around an object is to examine the forces acting on individual fluid particles and apply Newton's laws of motion. While there are many different types of forces acting on a fluid particle it is possible to neglect most of these, such as surface tension and gravity. In fact, for most practical flows the only relevant forces are due to pressure and friction. As a first step, we can also assume that there are no friction forces at work either. This is because in most flows friction is only significant in a very small region close to solid surfaces (the *boundary layer*). Elsewhere, friction forces are negligible.

We shall also assume the flow to be steady. In practice this means that we only consider situations where the overall flowfield does not change very quickly with time.

With these assumption we can now derive the rules governing fluid motion by considering the resultant pressure force acting on an individual fluid particle and applying Newton's second law, which states that force causes acceleration. As we

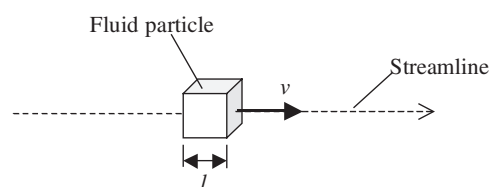


Figure 6. Fluid particle travelling along a straight line.

shall see, a force acting in the flow direction causes fluid particles to change their speed whereas a force acting normal to the flow direction causes streamline curvature (by 'particle' we refer to a very small but finite volume (or element) of the fluid, not individual molecules).

The 'truth' about Bernoulli

Imagine a fluid particle travelling along a straight line (but not at constant velocity) as shown schematically in figure 6. Let the x -direction be in the direction of motion. If the particle is in a region of varying pressure (a non-vanishing pressure gradient in the x -direction) and if the particle has a finite size l , then the front of the particle will be 'seeing' a different pressure from the rear. More precisely, if the pressure drops in the x -direction ($dp/dx < 0$) the pressure at the rear is higher than at the front and the particle experiences a (positive) net force. According to Newton's second law, this force causes an acceleration and the particle's velocity increases as it moves along the streamline. Conversely, if the pressure increases in the direction of the flow, the particle decelerates. *This means that if the pressure drops along a streamline, the velocity increases and vice versa.* Bernoulli's equation describes this mathematically (see the complete derivation in the appendix).

However, the fact is often overlooked that Bernoulli's equation applies only along a streamline. There is no explicit relationship between the pressure and velocity of neighbouring streamlines. Sometimes, all streamlines in a flow originate from a region where there is uniform velocity and pressure (such as a reservoir or a uniform free-stream) and in such a case it is possible to apply Bernoulli's equation throughout the flow. But in the 'demonstration' of Bernoulli's equation shown in figure 2 the air moving along the upper surface of the paper originates from the mouth of the person performing the experiment and the

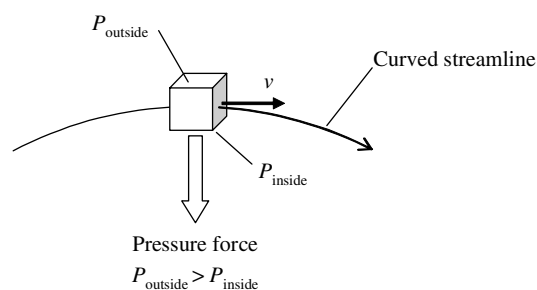


Figure 7. Fluid particle travelling along a curved streamline.

streamlines can be traced right back into this person's lungs. There is no connection with the 'streamlines' underneath the paper and Bernoulli's equation cannot be applied to compare the pressure in the two regions². In fact, the pressure in the air blown out of the lungs is equal to that of the surrounding air (and this is proved when blowing over a straight sheet of paper—it doesn't deflect towards the moving air).

Flow along curved streamlines

Next, examine a particle moving along a curved streamline as shown in figure 7. For simplicity we can assume that the particle's speed is constant³. Because the particle is changing direction there must exist a centripetal force acting normal to the direction of motion. This force can only be generated by pressure differences (all other forces are ignored), which implies that the pressure on one side of the particle is greater than that on the other. In other words, *if a streamline is curved, there must be a pressure gradient across the streamline, with the pressure increasing in the direction away from the centre of curvature.*

This relationship (derived mathematically in the appendix) between pressure fields and flow curvature is very useful for the understanding of fluid dynamics (although it doesn't have a name). Together with Bernoulli's equation, it describes the relationship between the pressure field and the flow velocity field. A good demonstration of this

² One might argue that the air blown over the top surface does eventually come to rest in the room and at that stage a connecting 'streamline' might be drawn towards the lower surface. However, in this case the flow is clearly affected by friction—this is what brings the flow to rest—and Bernoulli's equation only describes frictionless flows.

³ This in turn implies, according to Bernoulli's equation, that the pressure along the streamline is constant ($dp/dx = 0$).

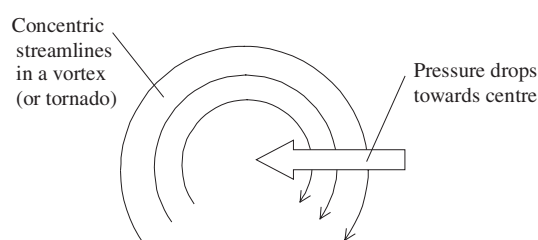


Figure 8. Pressure gradient across streamlines in a vortex.

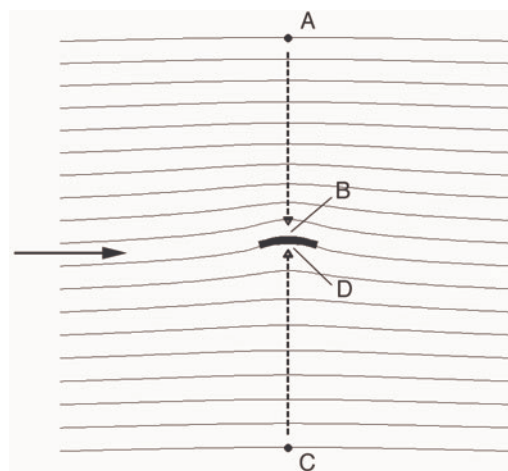


Figure 9. Streamlines around a lifting curved plate 'aerofoil'.

is a tornado (or any vortex, such as that seen in a bathtub). As sketched in figure 8, an idealized vortex consists of concentric circles of streamlines. The above relationship implies that there is a pressure gradient across these streamlines, with the pressure dropping as we approach the core. This explains why there are such low pressures in the centre of vortices (and why tornados 'suck' objects into the sky). In a real, three-dimensional, tornado, the streamlines are not circles but spirals which originate from somewhere far away where the air is at rest and at atmospheric pressure. Applying Bernoulli's equation to each of these streamlines shows that the velocities increase the closer we get to the vortex core (which is what we observe in nature)⁴.

⁴ At the very centre, the assumption of frictionless flow is no longer justified (because streamlines in opposite directions get very close to each other, creating a strong shear flow) and the above arguments no longer hold (the 'eye' of the storm has low flow velocities and low pressure).

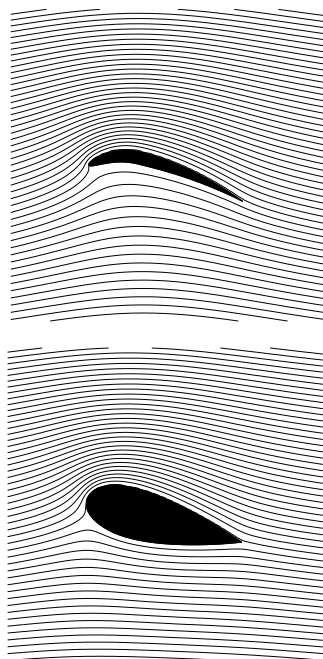
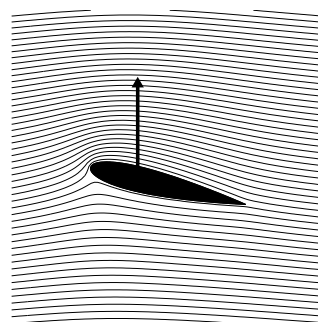


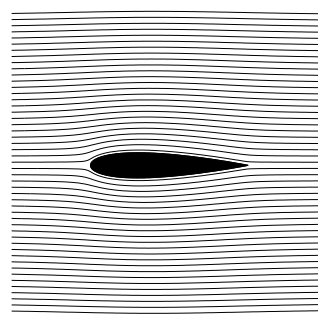
Figure 10. Simulated streamlines around thin and thick aerofoils.

Lift on aerofoils

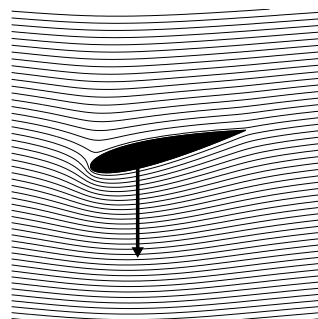
Now we can return to the original problem. Figure 9 shows a schematic sketch of the streamlines around the simplest lifting aerofoil—a curved plate. Far away the air is undisturbed by the presence of the wing, the pressure is atmospheric ($= p_{\text{atm}}$) and the streamlines are straight and horizontal. Now consider moving along a line from point A towards the surface, keeping on a path that is always perpendicular to the local streamline direction. Starting at A we note that the streamlines are straight and parallel and therefore there is no pressure gradient in the direction of the dashed line. However, closer to the aerofoil streamlines become increasingly curved and there must now be a pressure gradient across the streamlines. From the direction of curvature we note that the pressure drops as we move downwards. By the time we reach the aerofoil surface at B the pressure is noticeably lower than that at A ($p_B < p_{\text{atm}}$). In the same way we can imagine moving from C to D. Again, as we approach the aerofoil streamlines exhibit more and more curvature but this time the pressure increases towards the surface. At D the pressure is therefore greater than that at C ($p_D > p_{\text{atm}}$). Hence $p_B < p_D$ and this generates a resultant



(a)



(b)



(c)

Figure 11. Streamlines around a symmetrical aerofoil at various angles of attack. (a) Positive angle of attack: lift points up. (b) Zero angle of attack: no lift. (c) Negative angle of attack: lift points down.

pressure force on the aerofoil, acting upwards, i.e. lift.

From the above we learn that any shape that introduces curvature into the flowfield can generate lift. Aerofoils work because the flow follows the local surface curvature on the upper and lower surfaces. It is not necessary to consider frictional forces to explain lift, however; it is only due to the action of friction that streamlines take up the pattern we would intuitively expect, so strictly speaking lift would not be possible without friction.

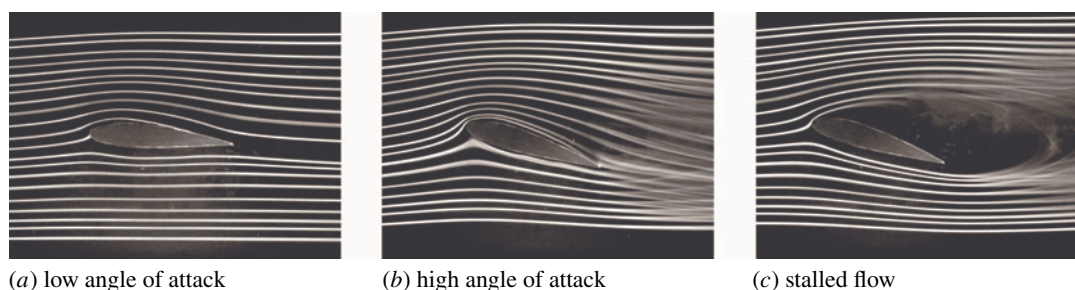


Figure 12. Streamlines around an aerofoil at increasing angle of attack.

Some observations resulting from this explanation

Following this line of argument it is possible to make some interesting observations. For example, consider the difference between the streamlines over a thin and a thick aerofoil as shown schematically in figure 10 (determined from a computer simulation). Despite the difference in thickness, both have similar flow patterns above the upper surface. However, there is considerable difference in the flow underneath. On the thin aerofoil the amount of flow curvature below the wing is comparable to that above it and we might conclude that the overpressure on the underside is just as large as the suction on the upper surface—the two sides contribute almost equally to the lift. In the case of the thick aerofoil, however, there are regions of different senses of curvature below the lower surface. This suggests that there will be areas with suction as well as areas with overpressure. In this case the lower surface does not contribute much resultant force and we can conclude that thin aerofoils are better at generating lift. This is generally true, and birds tend to have thin curved wings. Aircraft do not, because of the structural difficulties of making thin wings, and also because the volume contained in the wing is useful, e.g. for fuel storage.

A frequent question is how aircraft manage to fly upside down. To demonstrate this, figure 11 shows the streamlines over a symmetrical aerofoil at positive, zero and negative angles of attack. Just by judging the degree of flow curvature above and below the wing it can be seen that this aerofoil produces positive, zero and negative lift respectively. Negative lift (which would be required for flying upside down) is simply a question of the angle of attack at which the aircraft flies. Even non-symmetrical aerofoils can

generate negative lift, but they do require more severe negative angles of attack (because they still produce positive lift at zero angle of attack) which makes them less suitable for flying upside down.

This also demonstrates the significance of angle of attack, as seen again in figure 12. As the angle of attack of a wing increases, more flow curvature is introduced above the wing—compare (b) with (a)—and more lift is generated. However, at some point the flow is no longer capable of following the sharp curvature near the nose and it ‘separates’ from the surface. As a result the amount of streamline curvature above the wing has reduced considerably (see figure 12(c)), which causes a sharp drop-off in lift force. Unfortunately the process of flow detaching from the surface often happens instantaneously when the angle of attack is increased, making the loss of lift rather sudden and dangerous—this is called stall.

Conclusion

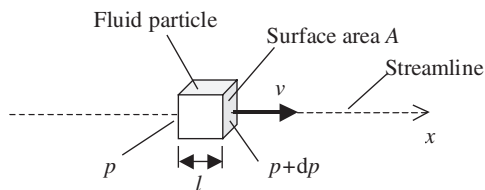
In this article I have attempted to give a ‘hands-on’ and correct explanation for Bernoulli’s equation, the relationship between streamline curvature and pressure, and lift. To explain lift it is not necessary to go through all of the above steps in the argument. Most students will be happy with the streamline pattern around a lifting wing (figure 1)—because it intuitively looks right—and this should be exploited. A short, but correct, explanation might start by discussing the existence of transverse pressure gradients in curved streamlines and applying this knowledge to the flowfield around an aerofoil in a similar manner to that shown in figure 9. This should explain why pressures on the two sides of an aerofoil are different. There is no need even to introduce Bernoulli’s equation or discuss the rather subtle significance of friction.

Acknowledgments

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Appendix

Pressure gradient along streamlines—Bernoulli's equation



In the above figure the cubic fluid particle experiences a pressure p behind and a slightly different pressure $p + dp$ in front. This causes an acceleration according to Newton's second law:

$$F = ma = m \frac{dv}{dt}.$$

Here the resultant pressure force is $F = -dp A$, which is negative because it points in the $-x$ direction. The mass of the fluid particle can be determined from its volume and the fluid density ρ : $m = lA\rho$.

The magnitude of the pressure change between front and back can be determined from the pressure gradient in the streamwise direction (dp/dx) and the size of the particle:

$$dp = l \frac{dp}{dx}.$$

Combining all of the above gives

$$-l \frac{dp}{dx} A = lA\rho \frac{dv}{dt}$$

which simplifies to

$$dp = -\rho \frac{dv}{dt} dx.$$

Noting that $dx/dt = v$, this can be rewritten as

$$dp = -\rho v dv.$$

Now we can integrate between two points (1 and 2) along a streamline to relate the pressure

difference between these points to the velocity difference:

$$\int_1^2 dp = - \int_1^2 \rho v dv$$

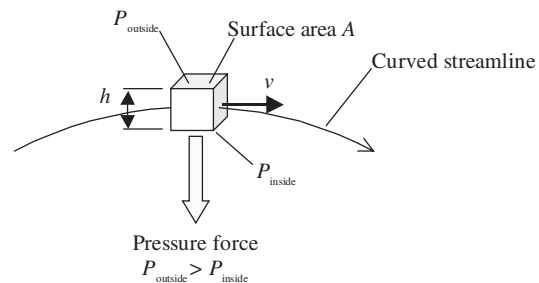
$$p_2 - p_1 = - \left(\rho \frac{v_2^2}{2} - \rho \frac{v_1^2}{2} \right)$$

which can be rearranged as

$$p_1 + \frac{\rho}{2} v_1^2 = p_2 + \frac{\rho}{2} v_2^2.$$

Because points 1 and 2 are arbitrary locations along the streamline, the above equation can be used to connect any two locations along a streamline—this is in essence Bernoulli's equation. Note that the streamline need not be straight.

Pressure gradient across curved streamlines



The centripetal force is $F = mv^2/R$ and we can define $p_{\text{inside}} = p$ and $p_{\text{outside}} = p + dp$. Similar to before we note that $m = \rho Ah$ and $dp = h(dp/dn)$, where n is the coordinate in the direction normal to the streamline (pointing away from the centre of curvature).

Combining all of the above yields

$$F = A dp = Ah \frac{dp}{dn} = \rho Ah \frac{v^2}{R}$$

which can be simplified to

$$\frac{dp}{dn} = \rho \frac{v^2}{R}$$

which expresses the pressure gradient *across* streamlines in terms of the local radius of curvature R and the flow velocity v . If a streamline is straight, $R \rightarrow \infty$ and $dp/dn = 0$. Therefore, there is no pressure gradient across straight streamlines.

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